МАТЕМАТИЧЕСКИЙ ЦЕНТР В АКАДЕМГОРОДКЕ

«Современные проблемы дифференциальных уравнений и их приложения»

Международная научная конференция

Ташкент, 23-25 ноября 2023 года

ТЕЗИСЫ ДОКЛАДОВ

ЧАСТЬ II

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НАЦИОНАЛЬНЫЙ УНИВЕРСИТЕТ УЗБЕКИСТАНА ИМЕНИ МИРЗО УЛУГБЕКА
ИНСТИТУТ МАТЕМАТИКИ ИМЕНИ В.И.РОМАНОВСКОГО АКАДЕМИИ НАУК РЕСПУБЛИКИ УЗБЕКИСТАН
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международной научной конференции на тему

СОВРЕМЕННЫЕ ПРОБЛЕМЫ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ И ИХ ПРИЛОЖЕНИЯ

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II

СОДЕРЖАНИЕ

The behavior of Trajectories of non-Volterra quadratic operators corresponding to permutations

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Let $E_m = \{1, \dots, m\}$ be a finite set and the set of all probability distributions on E_m

$$S^{m-1} = \{\mathbf{x} = (x_1, x_2, \dots, x_m) \in \mathbb{R}^m : \ x_i \geq 0, \text{ for any } i \text{ and } \sum_{i=1}^{m} x_i = 1\},$$

the $(m-1)$-dimensional simplex.

A map V of S^{m-1} into itself is called a *quadratic stochastic operator* (QSO) if

$$(V\mathbf{x})_k = \sum_{i,j \in E_m}^{m} p_{ij,k} x_i x_j, \tag{1}$$

for any $\mathbf{x} \in S^{m-1}$ and for all $k \in E_m$, where

$$p_{ij,k} \geq 0, \quad p_{ij,k} = p_{ji,k} \text{ for all } i,j,k \in E_m \text{ and } \quad \sum_{k=1}^{m} p_{ij,k} = 1. \tag{2}$$

A quadratic stochastic operator V defined by (1), (2) is called a Volterra operator if

$$p_{ij,k} = 0, \quad \text{for any } \ k \notin \{i,j\}, \quad i,j,k \in E_m.$$

Assume $\{\mathbf{x}^{(n)} \in S^{m-1} : n = 0, 1, 2, \dots\}$ is the trajectory (orbit) of the initial point $\mathbf{x} \in S^{m-1}$, where $\mathbf{x}^{(n+1)} = V(\mathbf{x}^{(n)})$ for all $n = 0, 1, 2, \dots$, with $\mathbf{x}^{(0)} = \mathbf{x}$. We denote by $\omega_V\left(\mathbf{x}^{(0)}\right)$ the set of ω- limiting points of the trajectory $\left\{V^n\left(\mathbf{x}^{(0)}\right)\right\}_{n=0}^{\infty}$.

The asymptotic behaviour of trajectories Volterra QSOs was analysed in [1].

Definition 1. A point $\mathbf{x} \in S^{m-1}$ is called a periodic point of V if there exists an n so that $V^n(\mathbf{x}) = \mathbf{x}$. The smallest positive integer n satisfying the above is called the prime period or least period of the point $\mathbf{x}$. A period-one point is called a fixed point of V.

Denote the set of all fixed points by $\mathrm{Fix}(V)$ and the set of all periodic points of (not necessarily prime) period n by $\mathrm{Per}_n(V)$. Evidently that the set of all iterates of a periodic point form a periodic trajectory (orbit).

We let a face of the simplex S^{m-1} be the set $\Gamma_\theta = \{\mathbf{x} \in S^{m-1} : \ x_k = 0, \ k \notin \theta \subset E_m\}$; let $\mathbf{e}_i = (\delta_{1,i}, \delta_{2,i}, \dots, \delta_{m,i}) \in S^{m-1}, \ i \in E_m$, denote the vertices of the simplex S^{m-1}, where $\delta_{i,j}$ is the Kronecker delta.

A permutation π of the set E_m is a *k–cycle* if there exists a positive integer k and an integer $i \in E_m$ such that k is the smallest positive integer such that $\pi^k(i) = i$, and π fixes each $j \in E_m \setminus \{i, \pi(i), \dots, \pi^{k-1}(i)\}$. The k-cycle of π is usually denoted $\left(i, \pi(i), \dots, \pi^{k-1}(i)\right)$. It is known that every permutation can be represented in the form of a product of cycles without common elements (i.e. disjoint cycles) and this representation is unique to within the order of the factors.

Let $\pi = \tau_1\tau_2\cdots\tau_q$ be a permutation of E_m, where τ_i, $i = 1,\ldots,q$ are disjoint cycles. Denote by $\mathrm{supp}(\pi)$ the support of the permutation π, i.e., $\mathrm{supp}(\pi) = \{j \in E_m : \pi(j) \neq j\}$ and let $|\mathrm{supp}(\pi)|$ be its cardinality. For a cycle τ_i we denote by $\mathrm{ord}(\tau_i)$ the order of the cycle and let $\mathrm{supp}(\tau_i) = \left\{j, \pi(j), \ldots, \pi^{\mathrm{ord}(\tau_i)-1}(j)\right\}$ be a set of elements which are generate the cycle. Clearly that $\mathrm{supp}(\pi) = \mathrm{supp}(\tau_1)\cup\cdots\cup\mathrm{supp}(\tau_q)$, where $\mathrm{supp}(\tau_i)\cap\mathrm{supp}(\tau_j) = \emptyset$, for any $i \neq j$.

Consider a non-Volterra QSO defined on a finite-dimensional simplex which has the form

$$V : \begin{cases} x_k' = \alpha x_{\pi(k)}^2 + \dfrac{2\alpha}{m-1}\sum\limits_{\substack{i,l=1:\\ i<l}}^{m-1} x_i x_l + (1+\alpha)x_{\pi(k)}x_m, \quad k \in E_{m-1}, \\ x_m' = x_m^2 + (1-\alpha)\left(\sum\limits_{i=1}^{m-1} x_i\right)^2 + (1-\alpha)x_m\left(\sum\limits_{i=1}^{m-1} x_i\right), \end{cases} \tag{3}$$

where $\alpha \in [0,1]$, $m > 3$ and π is a permutation on the set E_{m-1}.

Denote

$$\mathbf{x}_{\alpha,m} = \left(\frac{\alpha}{m-1}, \ldots, \frac{\alpha}{m-1}, 1-\alpha\right) \quad \text{and} \quad B = \widehat{B} \cup \widetilde{B}, \quad \text{where}$$

$$\widehat{B} = \left\{\mathbf{x} \in S^{m-1} : \ \mathbf{x}_i = (\alpha\delta_{1,i}, \ldots, \alpha\delta_{m-1,i}, 1-\alpha), \ i \in \mathrm{supp}(\pi)\right\},$$

$$\widetilde{B} = \left\{\mathbf{x} \in S^{m-1} : \ \mathbf{x}_i = (\alpha\delta_{1,i}, \ldots, \alpha\delta_{m-1,i}, 1-\alpha), \ i \in E_{m-1} \setminus \mathrm{supp}(\pi)\right\}.$$

Theorem 1. For the operator V (3) the following statements are true:

i) If $\alpha = 0$ then $\mathrm{Fix}\,(V) = \{\mathbf{e}_m\}$;

ii) If $0 < \alpha \leq 1$ then

ii_a)

$$\mathrm{Fix}\,(V) = \{\mathbf{e}_m\} \cup \{\mathbf{x}_{\alpha,m}\} \cup \begin{cases} B, & \text{if } \pi = Id, \\ \widetilde{B}, & \text{if } \pi \neq Id; \end{cases}$$

ii_b) If $\pi = Id$ then $\mathrm{Per}_n\,(V) = \emptyset$ for any $n \in \mathbb{N}$;

ii_c) Let $\pi \neq Id$ and let τ_i be a cycle of π with $t_i = \mathrm{ord}(\tau_i)$. Then we have

$$\mathrm{Per}_{t_i}\,(V) = \left\{\mathbf{x} \in \widehat{B} : \mathbf{x}_i = (\alpha\delta_{1,i}, \ldots, \alpha\delta_{m-1,i}, 1-\alpha), \ i \in \mathrm{supp}(\tau_i)\right\}.$$

Denote $C = \bigcup\limits_j \Gamma_{\{j,m\}}$ and $\mathbf{x}_\xi = \left(\xi_1, \xi_2, \ldots, \xi_{m-1}, 1-\alpha\right)$, where $\xi_j = \alpha\delta_{j,k}$, $j,k \in \mathrm{supp}(\tau_i)$.

Theorem 2. For the operator V the following statements are true:

i) if $\alpha = 0$, then $\omega_V\left(\mathbf{x}^{(0)}\right) = \{\mathbf{e}_m\}$ for any $\mathbf{x}^{(0)} \in S^{m-1}$;

ii) if $\alpha \in (0,1]$ then $\omega_V\left(\mathbf{x}^{(0)}\right) = \{\mathbf{x}_{\alpha,m}\}$ for any $\mathbf{x}^{(0)} \in S^{m-1} \setminus (C \cup \{\mathbf{e}_m\})$;

iii) if $\alpha \in (0,1]$, $\pi = Id$ then $\omega_V\left(\mathbf{x}^{(0)}\right) = \{\mathbf{x}^*\}$, $\mathbf{x}^* \in B$ for any $\mathbf{x}^{(0)} \in C \setminus \mathrm{Fix}\,(V)$;

iv) if $\alpha \in (0,1]$, $\pi \neq Id$ and $j \in E_{m-1} \setminus \mathrm{supp}(\pi)$ then $\omega_V\left(\mathbf{x}^{(0)}\right) = \{\mathbf{x}^*\}$, $\mathbf{x}^* \in \widetilde{B}$ for any $\mathbf{x}^{(0)} \in C \setminus \mathrm{Fix}\,(V)$;

v) if $\alpha \in (0,1]$, $\pi \neq Id$ and $j \in \text{supp}(\tau_i)$ then $\omega_V\left(\mathbf{x}^{(0)}\right) = \left\{\mathbf{x}_\xi, \mathbf{x}_\xi^1, \ldots, \mathbf{x}_\xi^{t_i-1}\right\}$ for any $\mathbf{x}^{(0)} \in C \setminus \text{Fix}\,(V)$.

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On dynamics of a non-volterra quadratic stochastic operator

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Let $E = \{1, \ldots, m\}$ be a finite set and the set of all probability distributions on E

$$S^{m-1} = \{\mathbf{x} = (x_1, x_2, \ldots, x_m) \in R^m : \ x_i \geq 0, \sum_{i=1}^m x_i = 1\}$$

be the $(m-1)$-dimensional simplex. A map V of S^{m-1} into itself is called a *quadratic stochastic operator* (QSO) if

$$(V\mathbf{x})_k = \sum_{i,j=1}^m P_{ij,k} x_i x_j \tag{1}$$

for any $\mathbf{x} \in S^{m-1}$ and for all $k = 1, ..., m$ where

$$P_{ij,k} \geq 0, \quad P_{ij,k} = P_{ji,k}, \quad \sum_{k=1}^m P_{ij,k} = 1. \tag{2}$$

In [1] developed the theory of Volterra QSOs. A *Volterra* QSO is defined by (1), (2) and with the additional assumption

$$P_{ij,k} = 0 \ \text{ if } \ k \notin \{i, j\}, \quad i, j, k \in E. \tag{3}$$

The trajectory $\left\{\mathbf{x}^{(n)}\right\}_{n \geq 0}$ of an operator V for a point $\mathbf{x} \in S^{m-1}$ is defined by $\mathbf{x}^{(n+1)} = V(\mathbf{x}^{(n)})$ for all $n = 0, 1, 2, \ldots$

Denote by $\omega_V\left(\mathbf{x}^{(0)}\right)$ the set of limit points of the trajectory $\left\{\mathbf{x}^{(n)}\right\}_{n \geq 0}$.

Definition 1. A point $\mathbf{x} \in S^{m-1}$ is called a *periodic point* of V if there exists an n so that $V^n(\mathbf{x}) = \mathbf{x}$. The smallest positive integer n satisfying the above is called the prime period or least period of the point x. A period-one point is called a *fixed point* of V, denote the set of all fixed points by $\text{Fix}\,(V)$.