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ABSTRACTS

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Section 11: Algebra, geometry and functional analysis

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The asymptotical behavior of trajectories of a non-Volterra quadratic operator

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Let $S^{m-1} = \{\mathbf{x} = (x_1, x_2, \dots, x_m) \in R^m : x_i \geq 0, \sum\limits_{i=1}^m x_i = 1\}$ be the $(m-1)$-dimensional simplex. A map V from S^{m-1} into itself is called a *quadratic stochastic operator* (QSO) if

$$(V\mathbf{x})_k = \sum_{i,j=1}^m p_{ij,k} x_i x_j, \quad k = 1, \dots, m, \tag{1}$$

for any $\mathbf{x} \in S^{m-1}$ and $p_{ij,k} \geq 0, \quad p_{ij,k} = p_{ji,k}$ for all $i, j, k, \quad \sum\limits_{k=1}^m p_{ij,k} = 1.$ (2)

Assume $\{V^n(\mathbf{x}^{(0)})\}_{n\geq 0}$ is the trajectory (orbit) of the initial point $\mathbf{x}^{(0)} \in S^{m-1}$, where $\mathbf{x}^{(n+1)} = V(\mathbf{x}^{(n)})$ for all $n = 0, 1, 2, \dots$ We denote by $\omega_V(\mathbf{x}^{(0)})$ the set of ω- limiting points of the trajectory $\{V^n(\mathbf{x}^{(0)})\}_{n\geq 0}$. One of the main problems in mathematical biology is investigation of the asymptotical behaviour of the $\{V^n(\mathbf{x}^{(0)})\}_{n\geq 0}$ for a given QSO, that is the description of the set $\omega(\mathbf{x}^{(0)})$ for any $\mathbf{x}^{(0)} \in S^{m-1}$ for a given QSO.

A QSO (1), is called Volterra if $p_{ij,k} = 0, \quad$ for any $\quad k \notin \{i, j\}, \quad i, j, k = 1, \dots, m$. The asymptotic behaviour of trajectories Volterra QSOs was deeply studied in [1].

We let $\mathbf{e}_i = (\delta_{1i}, \delta_{2i}, \dots, \delta_{mi}) \in S^{m-1}, \ i = 1, \dots, m$, denote the vertices of the simplex S^{m-1}, where δ_{ij} is the Kronecker delta.

Let us consider a non-Volterra QSO defined on the simplex S^2 which has the form

$$V: \begin{cases} x_1' = (1-\mu)x_1^2 + \gamma x_3^2 + (1-\beta-\mu)x_1x_2 + (1+\gamma-\mu)x_1x_3 + \gamma x_2x_3, \\ x_2' = (1-\mu)x_2^2 + (1+\beta-\mu)x_1x_2 + (1-\mu)x_2x_3, \\ x_3' = \mu x_1^2 + \mu x_2^2 + (1-\gamma)x_3^2 + 2\mu x_1x_2 + (1-\gamma+\mu)x_1x_3 + (1-\gamma+\mu)x_2x_3. \end{cases} \tag{2}$$

where $\gamma, \mu \in [0,1], \ \mu - 1 \leq \beta \leq 1 - \mu$. We denote $\mathbf{a} = \left(0, 1 - x_3^{(0)}, x_3^{(0)}\right)$, $\mathbf{b} = \left(1 - x_3^{(0)}, 0, x_3^{(0)}\right)$, $\mathbf{c} = \left(1 - x_2^{(0)}, x_2^{(0)}, 0\right)$, $\mathbf{x} = \left(1 - x_3^{(0)}, 0, x_3^{(0)}\right)$, $\mathbf{y} = \left(x_3^{(0)}, 0, 1 - x_3^{(0)}\right)$.

Theorem. For the QSO V the following statements are true:

$$\omega_V(\mathbf{x}^{(0)}) = \begin{cases} \{\mathbf{a}\}, & \text{if } \mathbf{x}^{(0)} \in S^2 \setminus Fix(V) \text{ and } \gamma = \mu = 0, \ \beta > 0, \\ \{\mathbf{b}\}, & \text{if } \mathbf{x}^{(0)} \in S^2 \setminus Fix(V) \text{ and } \gamma = \mu = 0, \ \beta < 0, \\ \{\mathbf{c}\}, & \text{if } \mathbf{x}^{(0)} \in S^2 \setminus Fix(V) \text{ and } \beta = \mu = 0, \ \gamma > 0, \\ \{\mathbf{e}_1\}, & \text{if } \mathbf{x}^{(0)} \in \Gamma_{13} \text{ and } \mu = 0, \ \beta\gamma > 0, \\ \{\mathbf{e}_2\}, & \text{if } \mathbf{x}^{(0)} \in S^2 \setminus \Gamma_{13} \text{ and } \mu = 0, \ \beta\gamma > 0, \\ \{\mathbf{e}_1\}, & \text{if } \mathbf{x}^{(0)} \in S^2 \setminus Fix(V) \text{ and } \mu = 0, \ \beta\gamma < 0, \\ \{\mathbf{e}_3\}, & \text{if } \mathbf{x}^{(0)} \in S^2 \setminus Fix(V) \text{ and } \gamma = 0, \ \mu > 0, \ \mu - 1 \leq \beta \leq 1 - \mu, \\ \{\widetilde{\mathbf{x}}\}, & \text{if } \mathbf{x}^{(0)} \in S^2 \setminus Fix(V) \text{ and } \beta = 0, \ \gamma\mu > 0, \ 0 < \gamma + \mu < 2, \\ \{\mathbf{x}, \mathbf{y}\}, & \text{if } \mathbf{x}^{(0)} \in S^2 \setminus Fix(V) \text{ and } \beta = 0, \ \gamma\mu > 0, \ \gamma + \mu = 2, \\ \{\widetilde{\mathbf{x}}\}, & \text{if } \mathbf{x}^{(0)} \in \Gamma_{13} \text{ and } \beta\gamma\mu > 0, \ \gamma(\beta - \mu) > \mu^2, \ \beta > \mu, \\ \{\widetilde{\mathbf{y}}\} & \text{if } \mathbf{x}^{(0)} \in S^2 \setminus \Gamma_{13} \text{ and } \beta\gamma\mu > 0, \ \gamma(\beta - \mu) > \mu^2, \ \beta > \mu, \\ \{\widetilde{\mathbf{x}}\}, & \text{if } \mathbf{x}^{(0)} \in S^2 \setminus Fix(V) \text{ and } \beta\gamma\mu > 0, \ \gamma(\beta - \mu) \leq \mu^2 \text{ or } \beta\gamma\mu < 0. \end{cases}$$

References

1. Ganikhodzhaev R.N., Quadratic stochastic operators, Lyapunov functions and tournaments. Sb. Math. 76(2), pp.489–506, 1993.